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Abstract

A simple relativistic model is suggested that elucidates the qualitative difference in valence quark distributions of bosons in ground and virtual states. The inelastic diffraction of highly virtual photon in DIS is discussed.

The unambiguous experimental evidence has been obtained [1] in DIS that the cross section of inelastic diffraction $\gamma^*(Q^2) + p \rightarrow hadrons + p$ is unexpectedly large even at $Q^2 \gg 1\ GeV^2$ and that its fraction in the total cross section of $\gamma^* p$ interaction is almost independent of Q^2. These results indicate the essential role which play fluctuations of highly virtual space-like photon into the hadronic states with sufficiently large transverse size. I would like to present a very simple and rather toy model which demonstrates that the transverse size of any highly virtual boson treated as a function of two variables, Q^2 and x (x being the energy fraction of a parton), is really expected to peak sharply near $x = m_q^2/Q^2$, approaching that of the boson on the mass shell. Therefore, this parton configuration should produce the diffraction pattern in the scattering of highly virtual boson similar to that of the real one. Being the striking qualitative effect, this feature of virtual particle interaction is expected to manifest itself (though, maybe, being attenuated numerically) within the frameworks of more realistic approaches. The consideration is based on the dispersion relation ideology [2] that parton distribution within the real or space-like boson (to be certain, I refer below to the photon) should be expressed by an integral over the time-like $q\bar{q}$-fluctuations in the relevant channel.[1] These fluctuations incorporate the low laying resonances ($\rho_0, \omega, etc.$) as well as the continuum background which contributes predominantly to the parton distribution at large Q^2. It is easy to estimate the weight function attached to a certain $q\bar{q}$-fluctuation. The contribution of $q\bar{q}$-fluctuation with the mass $M \geq 2m_q$, m_q being the quark mass, is proportional to the time Δt which this fluctuation spends in the state with four-momentum squared equal to $(-Q^2)$. In turn, for the

[1] A similar ideology has been exploited in the paper [3] for the evaluation of quark distribution at rather low values of Q^2.

photon with momentum $\vec{P}^2 \gg Q^2,\ \vec{P}^2 \gg M^2$

$$\Delta t \ \approx \ \frac{1}{\sqrt{\vec{P}^2 + M^2} - \sqrt{\vec{P}^2 - Q^2}} \ \simeq \ \frac{2|\vec{P}|}{M^2 + Q^2} \tag{1}$$

in the accordance with the energy-time uncertainty relation. It follows from Eq.(1) that parton distribution within the real photon $Q^2 = 0$ is saturated predominantly by resonances of the lowest masses only (namely, by ρ_0, ω and ϕ because the mass squared of the next resonance situated at the 1st daughter Regge trajectory is about four times larger), while for the photon with $Q^2 \gg 1\ GeV^2$ it is contributed essentially by the whole set of $q\bar{q}$-fluctuations with $M^2 \leq Q^2$. Thus, the calculation of quark distribution within virtual photon is reduced to summing up the properly weighed quark distributions in
$q\bar{q}$ -fluctuations with $2m_q^2 \leq M^2$. For the moment, the many particle (evolution) aspect of the problem is put aside. It will be briefly discussed below.

The simplest model is considered to describe $q\bar{q}$-fluctuations: two spinless particles of the same mass m_q (below referred as "quarks") interact via the confining potential

$$\begin{aligned} U = 0, \qquad & (x_1 - x_2)^2 + (y_1 - y_2)^2 + \frac{(z_1 - z_2)^2}{1 - v^2} \ < \ R^2, \\ U = +\infty, \qquad & \qquad\qquad \textit{outside of this ellipsoid}, \end{aligned} \tag{2}$$

where x_1, y_1, z_1 and x_2, y_2, z_2 are the quark coordinates, R is a constant, $1\ GeV^{-1} < R < 5\ GeV^{-1}$, and v is the velocity of their CMS along the axes z. The corresponding Shrodinger equation reads

$$\left(\sqrt{\vec{p}_1^{\,2} + m_q^2} + \sqrt{\vec{p}_2^{\,2} + m_q^2} - \frac{M}{\sqrt{1 - v^2}} \right) \psi(\vec{p}_1, \vec{p}_2) \ = \ 0 \tag{3}$$

The solution of Eq.(3) is obviously proportional to δ-function of the expression in brackets. In the limit $v \to 1,\ (|\vec{P}| \simeq |P_z| \to \infty)$

$$\sqrt{\vec{p}_1^{\,2} + m_q^2} + \sqrt{\vec{p}_2^{\,2} + m_q^2} \ \simeq \ \frac{m_T}{\sqrt{x(1-x)(1-v^2)}} \tag{4}$$

where $x = |\vec{p}_1| \ / \ (|\vec{p}_1| + |\vec{p}_2|)$ and m_T is the, so-called, transverse mass,

$$m_T \ = \ \sqrt{p_{1x}^2 + p_{1y}^2 + m_q^2} \ = \ \sqrt{p_{2x}^2 + p_{2y}^2 + m_q^2} \ \equiv \ \sqrt{p_T^2 + m_q^2}$$

Thus, at $Q^2 \gg 1\ GeV^2$ the wave function of highly virtual photon $\gamma^*(Q^2)$ is approximately equal to the integral over M of these δ-functions weighed in the accordance with Eq.(1), what leads to [2]

$$\psi_{\gamma^*} \;\simeq\; \frac{A|\vec{P}|\sqrt{1-v^2}}{Q^2 + m_T^2/x(1-x)}, \tag{5}$$

A being the normalization constant. Since the constraints $p_{1z} + p_{2z} = P_z$ and $\vec{p}_{1T} + \vec{p}_{2T} = 0$ are to be allowed for,

$$|\psi_{\gamma^*}|^2 \;\equiv\; \frac{dW}{d^3\vec{p}_1 d^3\vec{p}_2} \;\sim\; \frac{dW}{dm_T^2 d(p_{1z} - p_{2z})}$$

The standard relativistic kinematics [4] gives at $m_T^2 \ll \vec{p}_1^{\,2},\ m_T^2 \ll \vec{p}_2^{\,2}$

$$d(p_{1z} - p_{2z}) \;=\; \frac{m_T dx}{2x^{3/2}(1-x)^{3/2}\sqrt{1-v^2}}$$

and, finally, the normalized quark distribution reads as

$$\frac{dW}{dm_T dx} \;\simeq\; \frac{4Q}{\pi}\frac{m_T^2}{x^{3/2}(1-x)^{3/2}[Q^2 + m_T^2/x(1-x)]^2} \tag{6}$$

The most interesting feature of this distribution manifests itself, if one estimates the mean transverse size of the photon as the function of x and Q^2

$$< r(x,Q^2) > \simeq < \frac{1}{m_T} > = \int\limits_{m_q}^{\infty} \frac{dm_T}{m_T}\frac{dW}{dm_T dx} \simeq \frac{2x^{-1/2}(1-x)^{-1/2}}{\pi Q[1 + m_q^2/Q^2 x(1-x)]} \tag{7}$$

Herefrom, the value of $< r(x,Q^2) >$ is peaked sharply near the points

$$x = \frac{m_q^2}{Q^2} \qquad and \qquad (1-x) = \frac{m_q^2}{Q^2} \tag{8}$$

Irrespectively of Q^2 it is equal there to

$$r_{max} \;\simeq\; (\pi m_q)^{-1} \;\geq\; 1\ GeV^{-1} \tag{9}$$

[2]Strictly speaking, one has to sum up the projections of the solutions of Eq.(3) to the state with $J = 1$, however it does not affect the result, since $\cos\theta = (p_{1z} + p_{2z})\ /\ (|p_1 + p_2|) \to 1$

The $q\bar{q}$- -fluctuations of such a large size should exhibit the hadron-like strong interaction with the cross section which is caused by the degree of color descreening, i.e., is proportional to $< r^2(x, Q^2) >$. If the color descreening gets working effectively within the range $r_0 \leq r \leq r_{max}$ (in the accordance with Eq.(7), at $(r_{max}/r_0) \geq 2$ it is approximately equivalent to $(\pi r_0 m_q^2)^2/4Q^2 = x_1 \leq x(1-x) \leq x_2 = 4/(\pi r_0 Q)^2)$, then this cross section

$$\tilde{\sigma}_{\gamma^* p} \approx 4\pi\alpha \frac{8\pi}{\pi^2 Q^2} \int_{x_1}^{x_2} \frac{dx}{x} \approx 4\pi\alpha \frac{8}{\pi Q^2}[2\ln\frac{2}{\pi r_0 m_q} - 1] \geq \frac{4\pi\alpha}{Q^2} \qquad (10)$$

Thus, the contribution of the large size (soft) $\gamma^* p$ interaction turns out to be of the order (or even larger), than that of the "normal" DIS point-like $\gamma^* p$ interaction ($\simeq 4\pi\alpha/Q^2$). As to the former one, it is reasonable to expect that it should resemble closely the features of interaction of the real photon which exhibits a diffraction pattern with a certain non-vanishing (as the energy increases) fraction of photon inelastic diffraction into hadrons. The above model is far from being realistic and hardly can be used for obtaining the reliable quantitative results. However, its striking qualitative features can, most probably, stand against the necessary corrections which are of two kinds. First, in the framework of the two-particle approximation one has to allow for the direct quark-antiquark interaction via gluonic exchange and, therefore, to consider the linear confining potential instead of the potential wall of the infinite height. The corresponding corrections diminish the asymmetry in q and $\bar{q}$ energies and, hence, the contribution of the large size quark configurations in the γ^* wave function. Second, the many particle aspect of the problem is to be taken into account which is associated with the evolution equation or gluon bremsstrahlung (especially important is that produced by the slow quark or antiquark). These processes enlarge crucially the population of low energy partons and should lead to the strong enhancement of the large size effects. Thus, it seems reasonable to consider the distributions (6) and (7) as a quite natural initial conditions for the calculation of realistic parton distribution within highly virtual space-like photon.

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